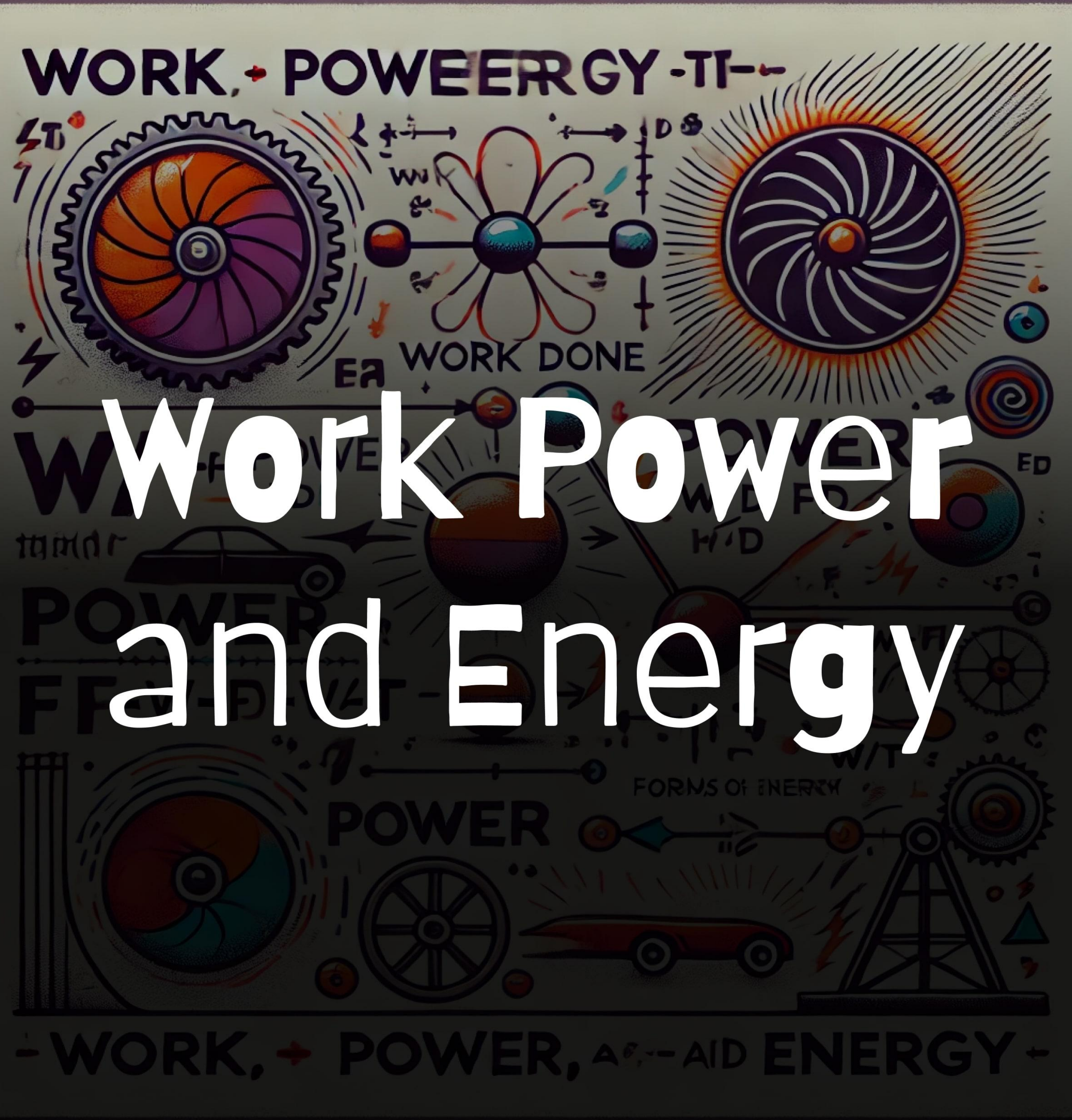
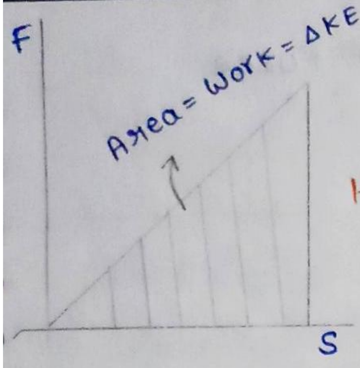
Work Power and Energy

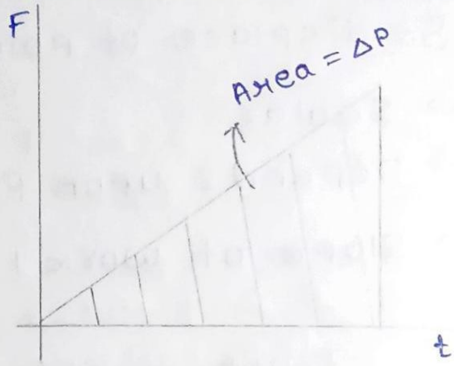


WORK POWER AND ENERGY



Force

How far (S) — How long (t)

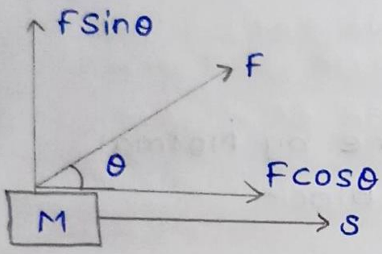


$F \cdot S = \text{work} = \Delta K \cdot E$

$F \cdot \Delta t = \Delta P$

$KE = \frac{P^2}{2m}$

Work Done By a Constant Force



$\text{Work} = \vec{F} \cdot \vec{S}$

$(F_x \hat{i} + F_y \hat{j} + F_z \hat{k}) \cdot (x\hat{i} + y\hat{j} + z\hat{k})$

$W = F_x x + F_y y + F_z z$

Work = (component of Force along dispm) . S

= (component of dispm along F) . F

$= F\cos\theta S$

$\omega = \vec{F} \cdot \vec{S}$

Work

By Constant Force → $\text{Work} = \vec{F} \cdot S$

$= FS\cos\theta$

By Variable Force → $dw = \int \vec{F} \cdot d\vec{s}$

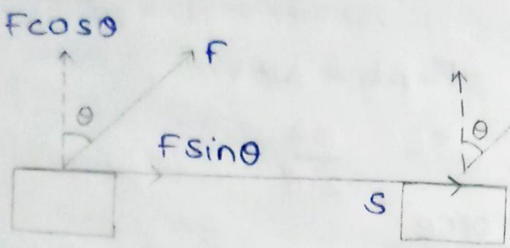
(Area of force dispm is work)

S → displace of point of application of force

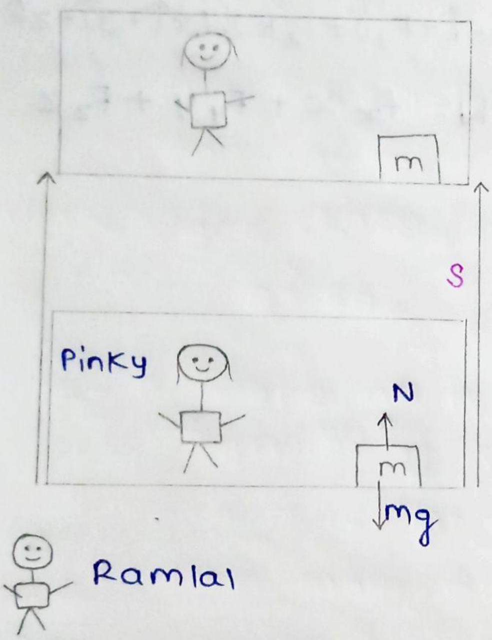
→ Scalar

→ Depends upon Frame

→ Slope of work / dispm is Force



$\text{Work} = (F\sin\theta)\,S$



Find work done by Normal force on the block

$W_{\text{by Normal}} = 0$ w.r.t Pinky bcoz dispm of block w.r.t Pinky is zero

$W_{\text{by Normal}} = +ve$

$= +NS$ w.r.t Ramlal

$\boxed{\text{Work} = \vec{F}\cdot\vec{S}}$

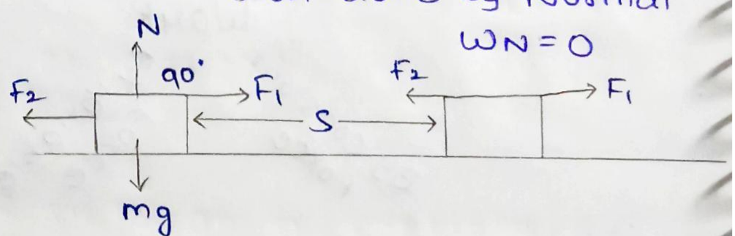
↳ always valid donot depend on Nature of motion.

$W_{net} = \vec{F}_{net}\cdot\vec{S}$ ✗

$W_{net} = F_1 \cdot S_1 + F_2 \cdot S_2 + \text{-----}$ ✓

Work done by Normal

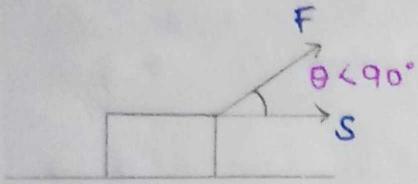
$W_N = 0$

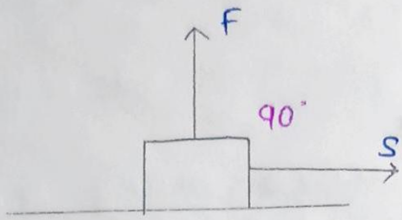


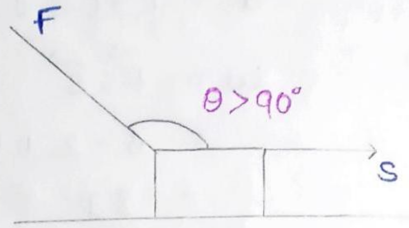
Work done by f_2 force in dispm is

$W_{F_2} = F_2 S(\cos 180^\circ)$

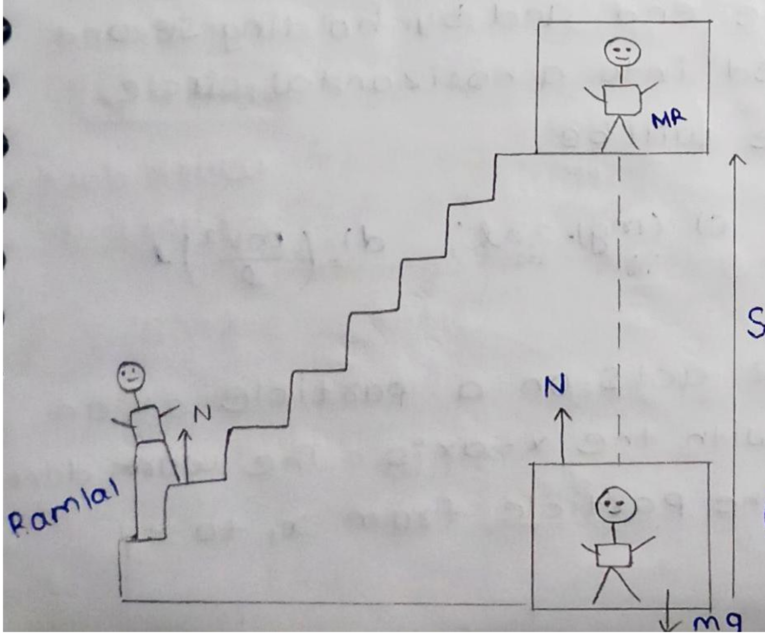
$= -F_2 S$







$W = FS\cos\theta$ $W = +ve$ Work may be +ve, −ve or zero	$W = FS\cos 90^\circ$ $W = 0$	$W = FS\cos\theta$ $W = -ve$
Work = +ve	Work = 0	Work = −ve
$0 \leq \theta < 90^\circ$ $W = +ve$ Speed ↑ Kinetic Energy ↑ Energy Inject	$\theta = 90^\circ$ $W = 0$ Kinetic Energy = const. Speed = constant	$90^\circ < \theta \leq 180^\circ$ $W = -ve$ Kinetic Energy ↓ Speed ↓



Work done on MR by Normal = +ve

Work done on Ramlal by Normal = 0

$(W_{MR})_{By\,N} = NS$

$(W_{Ramlal})_{By\,N} = 0$

Que - A force $\vec{F} = (2\hat{i} - \hat{j} - 4\hat{k})$ N displaces a particle upto $\vec{d} = (3\hat{i} + 2\hat{j})$ m. calculate work done.

$W = \vec{F} \cdot \vec{S}$

$= 6 - 2 + 4$

$= 8 J$

Que - A 2kg mass lying on a table is displaced in the horizontal direction through 50cm. The work done by the normal reaction will be

a) 0 b) 100 J c) 100 erg d) 10 J

Que - A Force of 10 N displaces an object by 10m. If work done is 50 J then direction of force make an angle with direction of dispm

$W = FS\cos\theta$

$50 = 10 \times 10 \cos\theta$

$\frac{1}{2} = \cos\theta$

$\theta = 60^\circ$

Que - A stone of mass m is tied to a string of length l at one end and by holding second end it is whirled into a horizontal circle, then work done will be

a) 0 b) $\left(\frac{mv^2}{l}\right) 2\pi l$ c) $(mg) \cdot 2\pi l$ d) $\left(\frac{mv^2}{l}\right) l$

Que - A force $f = kx^2$ acts on a particle at an angle of 60° with the x-axis. The work done in displacing the particle from x_1 to x_2 will be

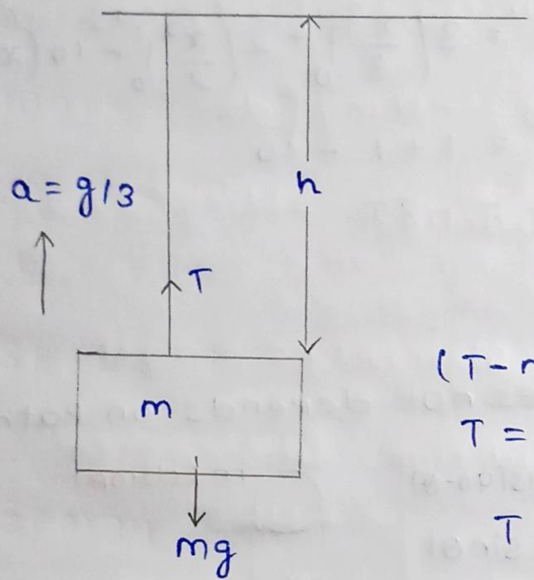
$$dw = \vec{F}\cdot d\vec{x}$$

$$dw = kx^2 dx \cos 60^\circ \rightarrow \int dw = \frac{k}{2}\int_{x_1}^{x_2} x^2\, dx$$

$$W = \frac{k}{2}\left[\frac{x^3}{3}\right]_{x_1}^{x_2}$$

$$= \frac{k}{6}\left(x_2^3 - x_1^3\right)$$

Que- A string is used to pull a block of mass m vertically up by a distance h at a constant acceleration g/3. The work done by the tension in the string is



$$W = Th \cos 0^\circ$$

$$= Th \times 1$$

$$= \frac{4mgh}{3}$$

$$(T - mg) = ma$$

$$T = mg + \frac{mg}{3}$$

$$T = \frac{4mg}{3}$$

Que- Block of mass m is pulled along a circular arc by means of a constant horizontal force F as shown. Work done by this force in pulling the block from A to B is

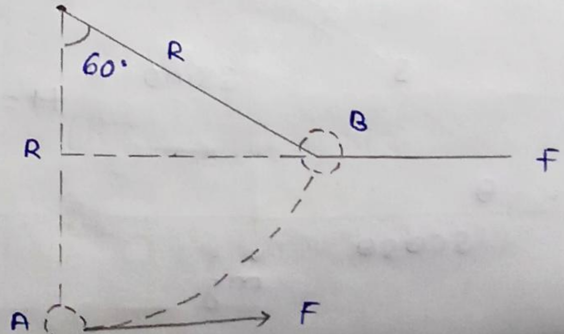
$$\sin\theta = \frac{x}{R}$$

$$x = R\sin\theta$$

$$W = Fx = FR\sin\theta$$

$$W = FR\sin 60^\circ$$

$$= \frac{FR\sqrt{3}}{2}$$



Que- work done by frictional force

a) Is always negative b) Is always positive

c) Is zero (d) may be +ve, -ve & 0

Que- A Particle move along x-axis from $x=0$ to $x=1$ m under the influence of a force given by $F = 3x^2 + 2x - 10$. Work done in the process is

variable

$$dw = F\,dx$$

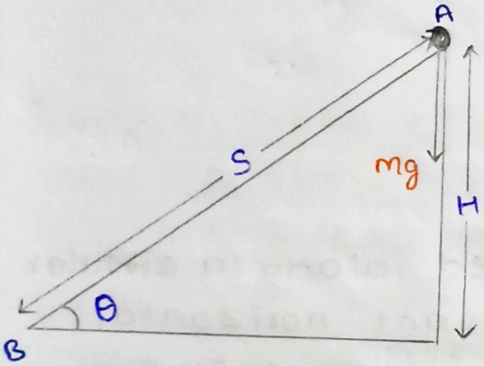
$$dw = (3x^2 + 2x - 10)\,dx$$

$$\int dw = \int_0^1 3x^2 dx + \int_0^1 2x\,dx - \int_0^1 10\,dx = 3\left(\frac{x^3}{3}\right)_0^1 + 2\left(\frac{x^2}{2}\right)_0^1 - 10(x)_0^1$$

$$= 1 + 1 - 10$$

$$= -8\,J$$

Work done by Gravity → does not depends on Path



$$W_g = mgS\cos(90-\theta)$$

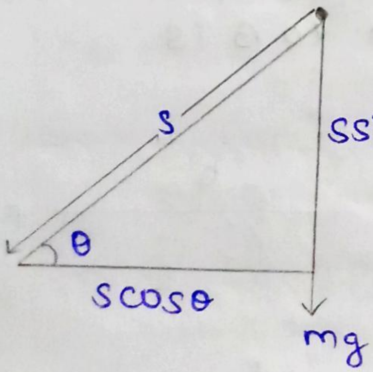
$$= mgS(\sin\theta)$$

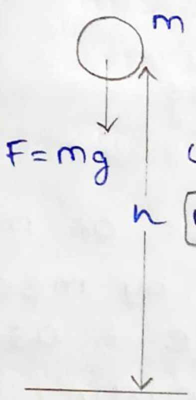
$$W_g = mgH$$

$$W_g = mg\begin{pmatrix}\text{vertical}\\ \text{dispm}\end{pmatrix}$$

$$\sin\theta = \frac{H}{S}$$

$$H = S(\sin\theta)$$



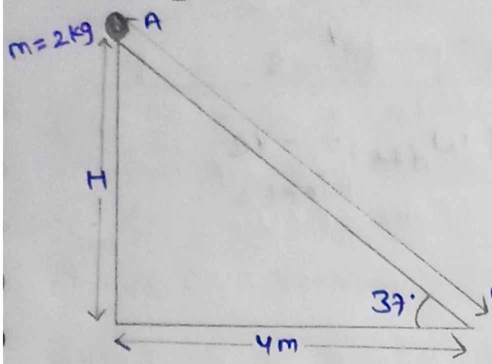


$$W = mgh\cos 0°$$

$$\boxed{W = +mgh}$$

work done by gravity

Find work done by gravity when object moves from A to B



$W = mgH$

$= 2 \times 10 \times 3$

$= 60\,m$

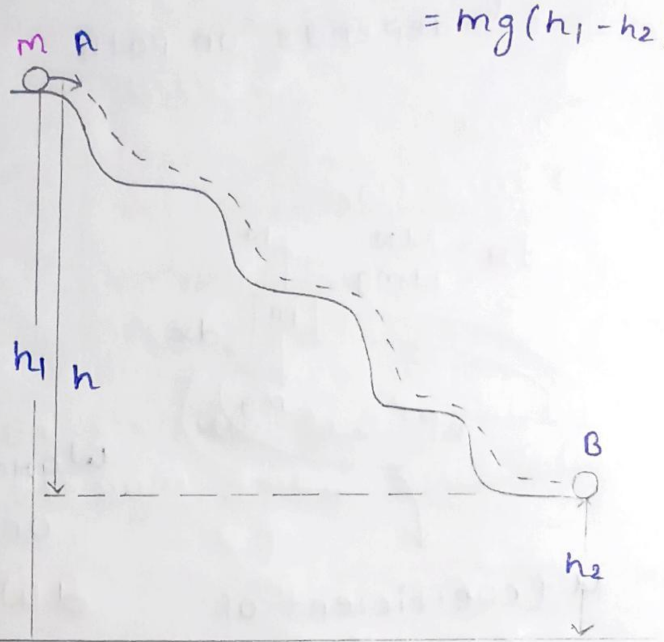
$\tan 37° = \frac{H}{4}$

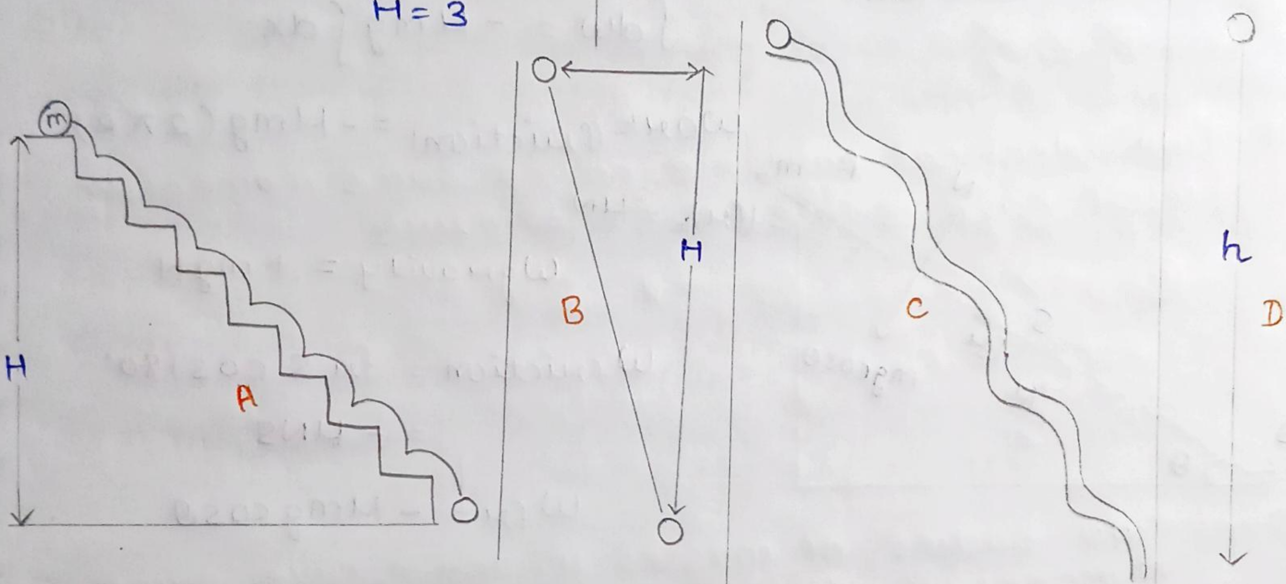
$\frac{3}{4} = \frac{H}{4}$

$H = 3$

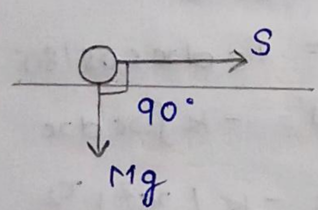
$W_{gravity} = mg(h)$

$= mg(h_1 - h_2)$

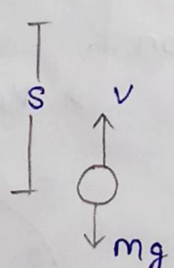




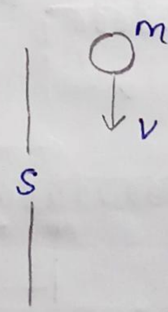
$W_{gravity} = W_A = W_B = W_C = W_D$



$W_{gravity} = 0$



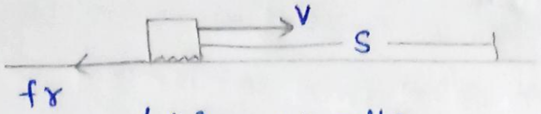
$W_{gravity} = -ve$



$W_{gravity} = +ve$

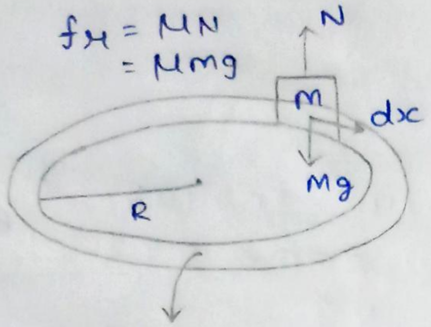
Work done by Friction → Friction always acts along the Path.

↳ depends on Path



$(W_{fr})_{kinetic} = -ve$

$f_r = \mu N$
$= \mu mg$



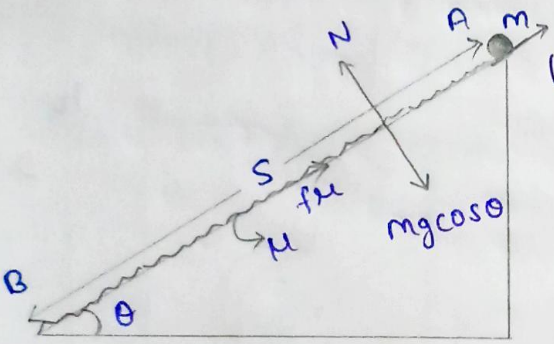
μ (coefficient of friction)

$W_{gravity}$ in one Rotation $= 0$

$dW_{friction} = f_r\, dx \cos 180^\circ$

$\int dw = -\mu mg \int dx$

$Work_{(friction)} = -\mu mg(2\pi R)$



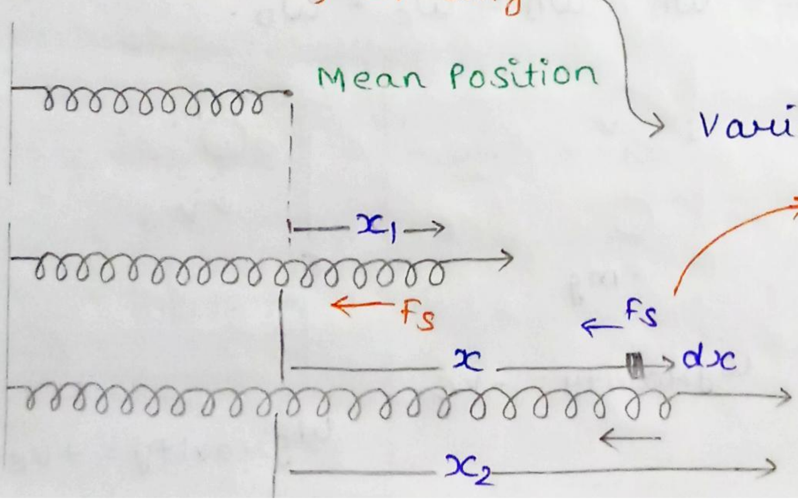
$(f_r)_k = \mu N$

$W_{gravity} = +mgH$

$W_{friction} = f_r\, S \cos 180^\circ$
$= -\mu NS$

$W_{fr} = -\mu mg \cos\theta$

Work done by Spring → Variable Force



$dw = f_s\, dx \cos 180^\circ$

$\int dw = -k \int x\, dx$

$W = -k \left(\frac{x^2}{2}\right)_{x_1}^{x_2}$

$W_{\text{by spring force}} = -\frac{1}{2}k[x_2^2 - x_1^2]$

Work done by spring in elongation from x_1 to x_2

Work done in compression From x_1 to x_2

$W = \frac{1}{2}k[x_2^2 - x_1^2]$

Work done by spring in elongation $= -ve$

work done by spring in compression $= -ve$

Que - If $x_1 = 0$ (mean) to $x_2 = x$ (elongation)

$W_{\text{spring}} = -\frac{1}{2}k[x^2 - 0^2]$

$W = -\frac{1}{2}kx^2$

Que - If work done by spring is W when elongate 2m then find work done in further elongation by 2m

case - I $\quad W_{sf} = -\frac{1}{2}k(2)^2 = -\frac{4k}{2} = W$

case - II

$W_{sf} = -\frac{1}{2}k[(4)^2 - (2)^2]$

$= -\frac{1}{2}k[16-4] = -\frac{12k}{2}$

$= -$

Momentum → motion contained in a body.

↳ Vector (dirn along velocity)

$\vec{P} = m\vec{V} \leftarrow$ Velocity

$|\vec{P}| = mv \rightarrow$ Speed

K.E. = (Energy stored due to motion)

$K.E. = \frac{1}{2} m(v^2)$

Scalar ↑ ← speed

No direction

$K.E. = \frac{1}{2} \frac{mv^2 \times m}{m} = \frac{p^2}{2m}$

↓

Relation b/w K.E. and magnitude of momentum.

Two identical particle having same K.E. must have same momentum. ⇒ False

(m) → v   (m) ↑ v

$K.E_1 = K.E_2$

$\vec{P_1} \neq \vec{P_2}$

Two identical particle having same momentum must have same K.E. ⇒ True

(m) → v   (m) → v

$K.E_1 = K.E_2$

$\vec{P_1} = \vec{P_2}$

Que- Two bodies of masses m_1 and m_2 have same momentum. The ratio of their K.E. is

a)

$K.E = \frac{1}{2} mu^2$

$K.E. \propto m$

$\frac{K.E_1}{K.E_2} = \frac{m_1}{m_2}$

u = constant

$K.E. = \frac{p^2}{2m}$

$K.E. \propto \frac{1}{m}$

$\frac{K.E_1}{K.E_2} = \frac{m_2}{m_1}$ P = same

Que- Two bodies with K.E. in the ratio of 4:1 are moving with equal linear momentum. The ratio of their masses is

$$\frac{K.E_1}{K.E_2} = \frac{4}{1} \qquad (P = \text{same})$$

Que- A stationery Particle explodes into two particles of masses m_1 and m_2 which move in opposite directions with velocities v_1 and v_2. The ratio of their kinetic energies $\frac{E_1}{E_2}$ is

Same as Gun-Bullet

(m) $\longrightarrow$ $\overset{\leftarrow}{P}$ (m₁) (m₂) $\vec{P}$

rest

$P = 0$ $F_{ext} = 0$ $P_f = 0 = \vec{P_1} + \vec{P_2}$

Que- Two bodies of masses m_1 and m_2 are moving with same kinetic Energy. If P_1 and P_2 are their respective momentum, the ratio P_1/P_2 is equal to

$$P = mv$$

$$\frac{P_1}{P_2} = \frac{m_1}{m_2}$$

$$K.E. = \frac{P^2}{2m}$$

$$P = \sqrt{2mK.E.}$$

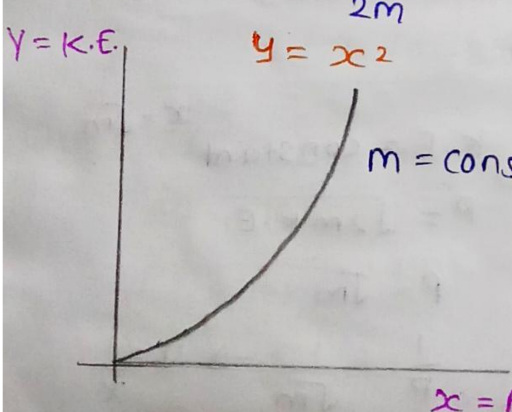
$(K.E = \text{Same})$

$$P \propto \sqrt{m}$$

Graph b/w P and K.E.

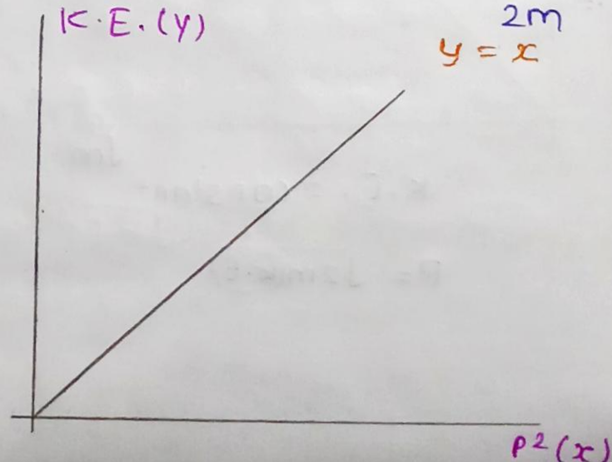
$$K.E. = \frac{P^2}{2m}$$

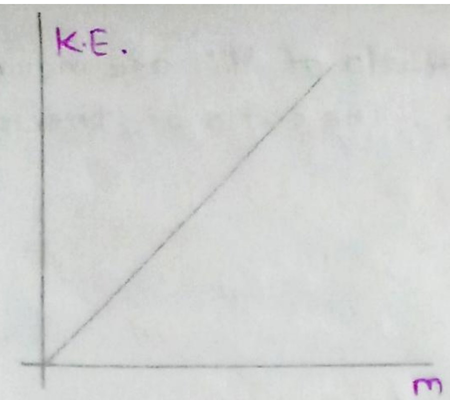
$$y = x^2$$



$$K.E. = \frac{P^2}{2m}$$

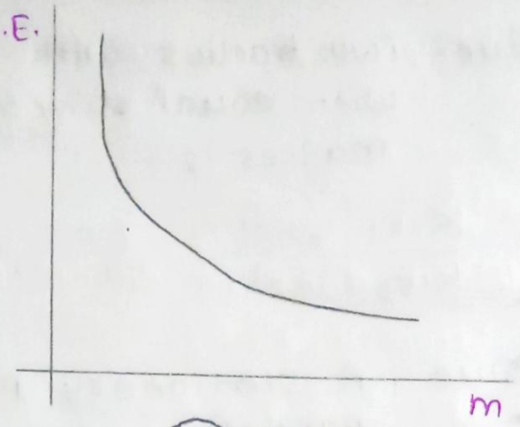
$$y = x$$





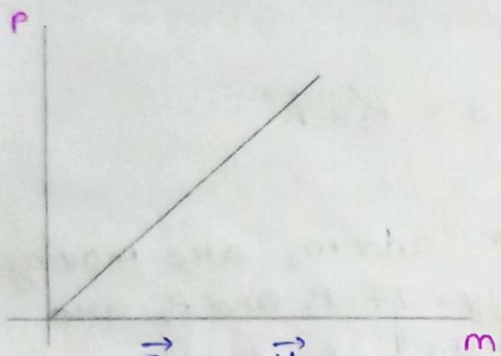
$K.E. = \frac{1}{2} m v^2$

$K.E. \propto m$



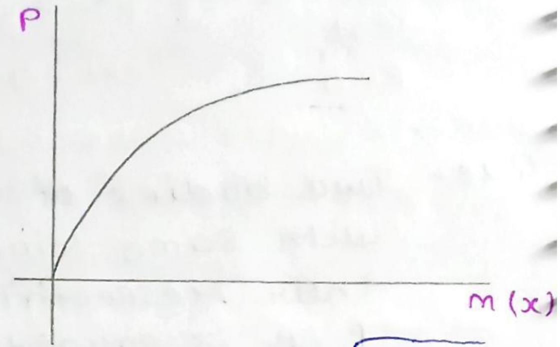
$K.E. = \frac{p^2}{2m} \rightarrow$ Same

$K.E. \propto \frac{1}{m}$



$\vec{P} = m\vec{v}$

$P \propto m$

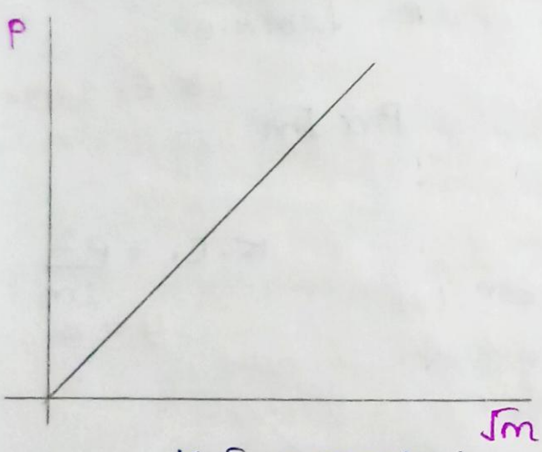


$K.E. = \frac{p^2}{2m}$

$p = \sqrt{2mK.E.}$

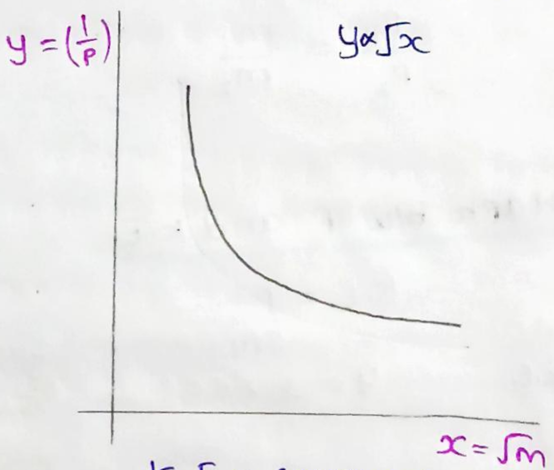
$p \propto \sqrt{m}$

$y \propto \sqrt{x}$



K.E. = constant

$P = \sqrt{2m K.E}$



K.E = constant

$P = \sqrt{2mK.E.}$

$P = \sqrt{m}$

$\frac{1}{P} = \frac{1}{\sqrt{m}} \rightarrow y = \frac{1}{x}$

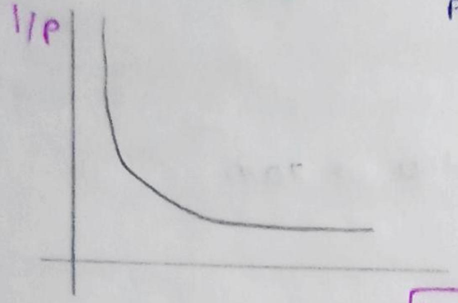
Graph b/w $\sqrt{K.E.}$ and $(1/P)$

$m =$ constant

$P = \sqrt{2m\,K.E.}$ → $P = \sqrt{K.E.}$

$2m$ ↓ constant

$\frac{1}{P} = \frac{1}{\sqrt{K.E.}}$ $\left(y = \frac{1}{x}\right)$



% age Change

Small Change

Use Error Analysis

$K.E. = \frac{P^2}{2m}$ ✗

$\frac{\Delta K.E}{K.E} = 2\frac{\Delta P}{P} + \frac{\Delta m}{m}$

If $m =$ constant :

$\left[100 \times \frac{\Delta K.E}{K.E}\right] = 2\left[\frac{\Delta P}{P} \times 100\right]$

$P = \sqrt{2m\,K.E.}$

$2m$ ↓ constant

$P = \sqrt{K.E.}$

$100 \times \frac{\Delta P}{P} = \frac{1}{2}\frac{\Delta K.E}{K.E} \times 100$

upto 5%.

Large change

$\%\ \text{change} = \frac{x_f - x_i}{x_i} \times 100$

% change in P $= \frac{P_f - P_i}{P_i} \times 100$

Que - If % change in K.E. is 3% then find %age change in momentum

$P = \sqrt{2m\,K.E.}$

$P = \sqrt{K.E.}$

$100 \times \frac{\Delta P}{P} = \frac{1}{2}\frac{\Delta K.E.}{K.E.} \times 100$

$= \frac{1}{2} \times 3 = 1.5\%$

Que- K.E. of a body is increased by 44%. What is the percent increase in the momentum.?

$$K.E_f = 144\% \text{ of } K.E_i$$

$$= \frac{144}{100} \times K.E_i$$

$$P_f = \sqrt{K.E_f}$$

$$P_f = \sqrt{\frac{144}{100} K.E_i} = \sqrt{\frac{12}{10}} \times 100 = 20\%$$

If K.E. is Increase by 44% then $K.E_f = 144\% \; K.E_i$

$$= \frac{144}{100} \times K.E_i$$

If K.E. is decrease by 44% then $K.E_f = 56\% \; K.E_i$

If K.E. is increase by 30% then $K.E_f = 130\% \; K.E_i$

If K.E is decreases by 17% then $K.E_f = 83\% \; K.E_i$

If K.E. is changed to 30% then $K.E_f = 30\% \; K.E_i$

Que- If K.E. of a body is increased by 300% then percentage change in momentum will be

$$K.E_f = 400 \times K.E_i$$

$$K.E_f = \frac{400}{100} \times K.E_i$$

$$P_f = \sqrt{K.E_f} = \sqrt{4} \times 100$$

$$= 2 \times 100 = 200\%$$

Que- When momentum of a body increases by 200%. Its KE increases by

$\vec{P_f} = 300\% \cdot P_i$

$= \frac{300}{100} P_i = 3P_i$

$K.E_f = \frac{P^2}{2m} = (3P_i)^2 = 9$

$= 900\%$

$\% K.E. = \frac{K.E_f - K.E_i}{K.E_i} \times 100$

$= \frac{\frac{P_f^2}{2m} - \frac{P_i^2}{2m}}{\frac{P_i^2}{2m}} \times 100$

$\frac{9P_i^2 - P_i^2}{P_i^2} \times 100$

$= 8 \times 100 = 800\%$

Que- If K.E is decrease by 19% then %age chang in momentum

$K.E_f = 81\% \cdot K.E_i$

$= \frac{81}{100} K.E_i$

$P_f = \sqrt{K.E_f}$

$= \sqrt{\frac{81}{100} K.E_i}$

$= \frac{9}{10} \times 100 = 90\%$ → decreases by 10%

Work Energy Theorem:

→ Valid in all frame

→ Valid for all type of motion

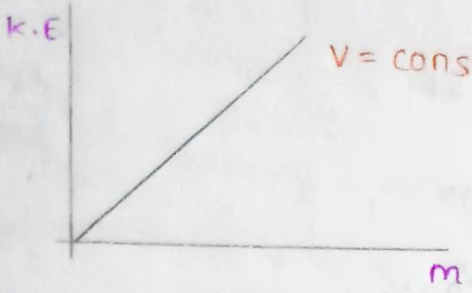
→ Valid for all type of force

→ Valid in all chapter

$W_{C.F} + W_{N.C.F} + W_{int.F} + W_{ext.F} + W_{Pseudo} \ldots = \Delta K.E.$

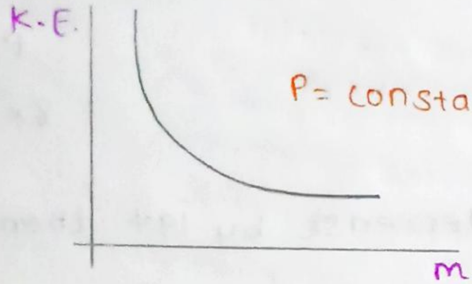
$W_{all\ Force} = \Delta K.E.$

$K.E. = \frac{1}{2} m v^2$



V = constant

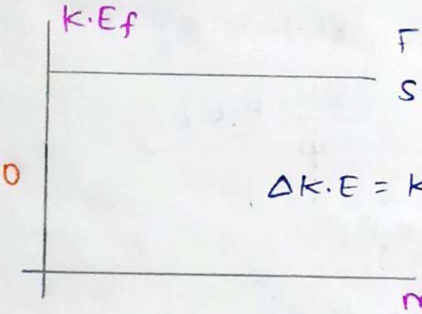
$K.E. = \frac{p^2}{2m}$



P = constant

$\Delta K.E = W_{Total} = \vec{F} \cdot \vec{S}$

If $K.E_i = 0$

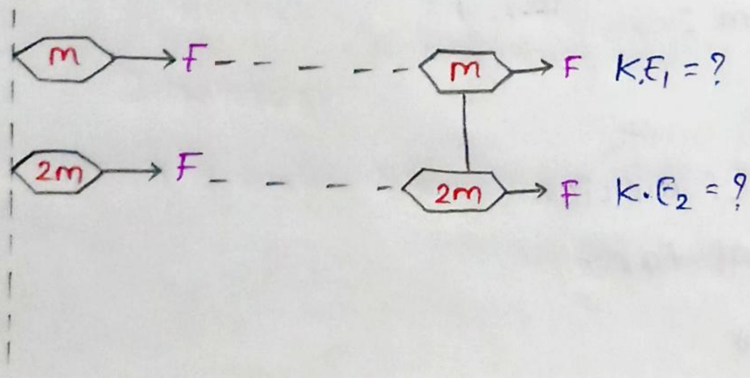


F = constant
S = constant
→ Same

$\Delta K.E = K.E_f - K.E_i = W$
$= F \cdot S$

Work Energy Theorem

Work done by all Force is equal to change in K.E.



For Ist object

$W_{all} = K.E_f - \cancel{K.E_i}^{\,0}$

$FS = K.E_f \quad \text{--- (i)}$

For IInd object

$FS = K.E_2 \quad \text{--- (ii)}$

$\frac{K.E_1}{K.E_2} = \frac{1}{1}$

Que - Under the action of a force, a 2kg body moves such that its position x as a function of time t is given by $x = t^2/3$, where x is in meters and t in seconds. The work done by the force in first two seconds is

$t_i = 0 \qquad t_f = 2$

$W = F \cdot S \quad \times$

$W = \Delta K.E = K.E_f - K.E_i$

$W = \frac{1}{2} m \left[v_f^2 - v_i^2 \right]$

$= \frac{1}{2} 2 \left[\left(\frac{4}{3}\right)^2 - 0^2 \right]$

$= \frac{16}{9}$ Joule

$\frac{dx}{dt} = v = \frac{1}{3} 2t$

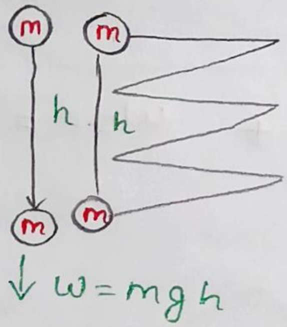
$= \frac{2}{3} t$

$v_i = 0$

$v_f = \frac{2 \times 2}{3} = \frac{4}{3}$

Work done depends on Frame of Reference

Work done by gravity does not depend on Path



But work done by Friction depends on Path

$\boxed{\Delta U = - W_{C.F}}$

Change in P.E. is just a name given to -ve work done by C.F

$\Delta U = - \vec{F} \cdot \vec{S} = - \int F \cdot ds$

$dU = - F \cdot ds$

$\vec{F}_{C.F} = - \left(\frac{dU}{ds} \right)$

$\boxed{\text{Work Energy Theorem}}$ → always Valid
→ Baap of all laws

$W_{\text{all Forces}} = \Delta K.E.$

$W_{CF} + W_{NCF} = \Delta K.E.$

$\boxed{\text{If } W_{NCF} = 0\text{; NCF may or may not be acting but } W_{NCF} \text{ is zero}}$

$W_{C.F} + W_{NCF} = \Delta K.E.$

$-\Delta U = \Delta K.E.$

$\Delta K.E + \Delta U = 0$

$\Delta(K.E + U) = 0$

$K.E + U = \text{constant}$

$\boxed{K.E_i + U_i = K.E_f + U_f}$

→ conservation of mechanical Energy Theorem

→ Not always valid only valid when $W_{NCF} = 0$

→ This is special case of work Energy Theorem

→ Beta of work Energy Theorem

$\boxed{\Delta U = -W_{C.F}}$ असली

↳ mother of all laws

→ always valid

Work Energy Theorem

$W_{CF} + W_{NCF} = \Delta K.E$

If $KE = \text{constant}$ (slowly)

$W_{CF} + W_{NCF} = 0$

$-\Delta U + W_{NCF} = 0$

$\boxed{W_{NCF} = \Delta U}$ नकली

If $K.E = \text{constant}$ and NCF is working against C.F.

Work Energy Theorem + $W_{C.F} = -\Delta U$

$W_{C.F} + W_{NCF} = \Delta K.E$

↳ always valid

→ Baap of all laws

$W_{C.F} = -\Delta U$

↳ always Valid

→ mother of all laws

$(K.E + U)_{\text{initial}} = (K.E + U)_{\text{Final}}$

C.O.M.E

→ only valid when $W_{NCF} = 0$

→ Beta

$W_{NCF} = \Delta U$

↳ only valid when $K.E = \text{constant}$ and NCF is acting against C.F.

→ Beti

हमारा Force → NCF   C.O.M.E

जहाँ C.O.M.E वहाँ हम नहीं
जहाँ हम, वहाँ C.O.M.E. नहीं

Que- Work done In bringing object from A to B slowly is 40 J then find Potential energy at B if $V_A = -30$ J

* → W_{NCF}

$W_{NCF} = \Delta U$ (Beti Law)

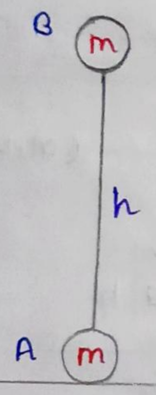
$40 J = U_B - U_A$

$40 J = U_B - (-30 J)$

$40 - 30 = U_B$

$U_B = 10$ J

Gravitational Potential Energy



$-W_{CF} = \Delta U$

$A \longrightarrow B$

$-[-mgh] = U_B - U_A$

$[U_B - U_A] = mgh$

If $U_A = 0$ (Let)

↳ A (ref. Point)

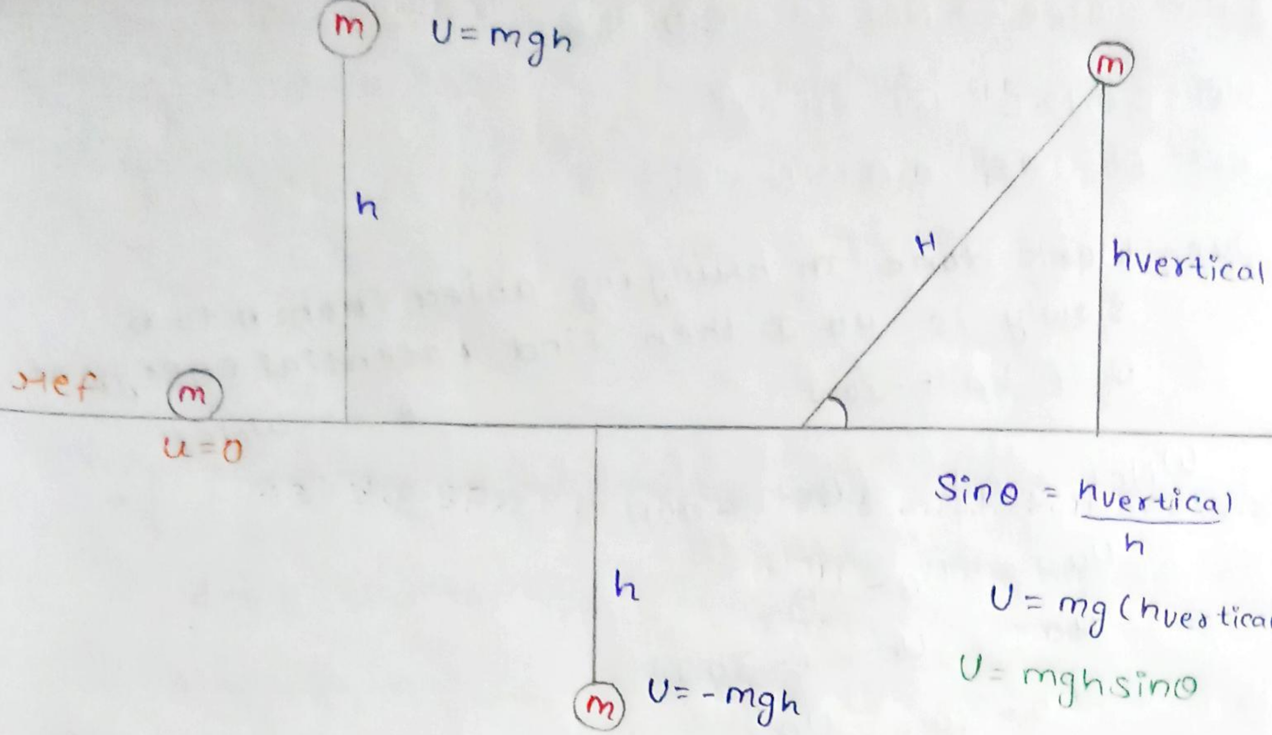
$U_B = mgh$

If ref^n is at B

$U_B = 0$ (Let)

$0 - U_A = mgh$

$U_A = -mgh$

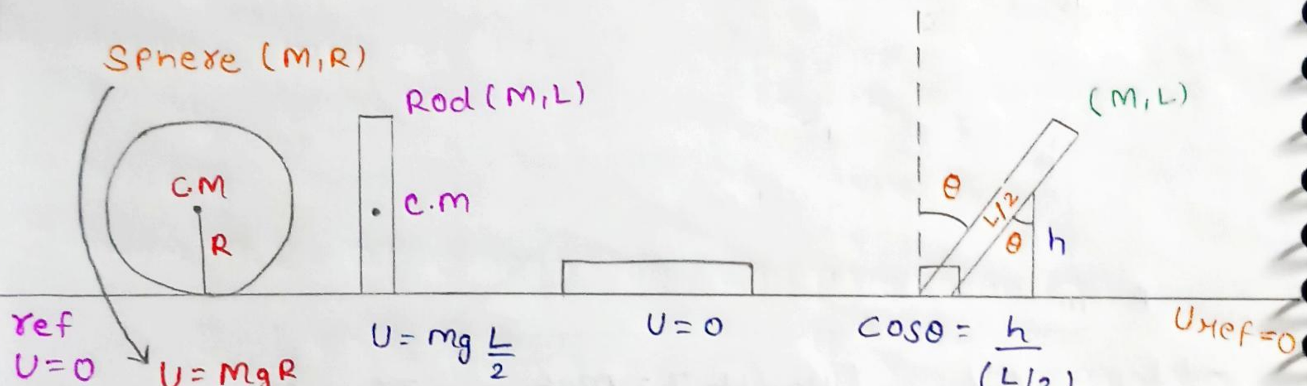


$$\sin\theta = \frac{h_{vertical}}{h}$$

$$U = mg(h_{vertical})$$

$$U = mgh\sin\theta$$

Work Done By Gravitation Field Only Potential



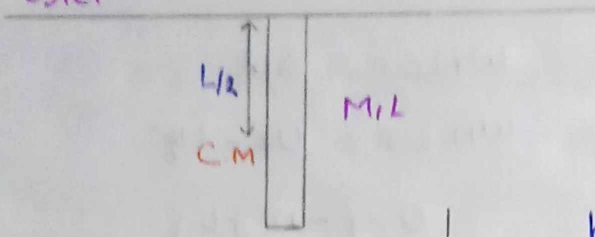
$$\cos\theta = \frac{h}{(L/2)}$$

$$h = \frac{L}{2}\cos\theta$$

$$U = mgh = \frac{mgL\cos\theta}{2}$$

MR* If $\theta = 90^\circ$, $U = 0$ | $\theta = 0^\circ$, $U = mg\frac{L}{2}$

$U_{ref} = 0$

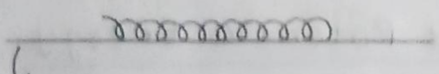


$U = -mg\frac{L}{2}$

half of length is hand over the table.

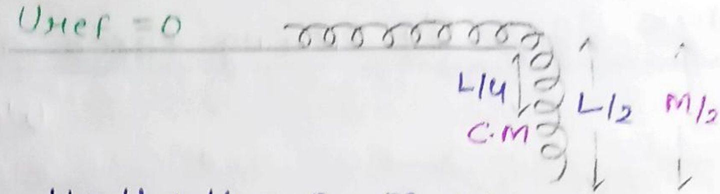
chain → (M, L)

chain (M, L)



$U_{ref} = 0$ $\quad U = 0$

$U_{ref} = 0$

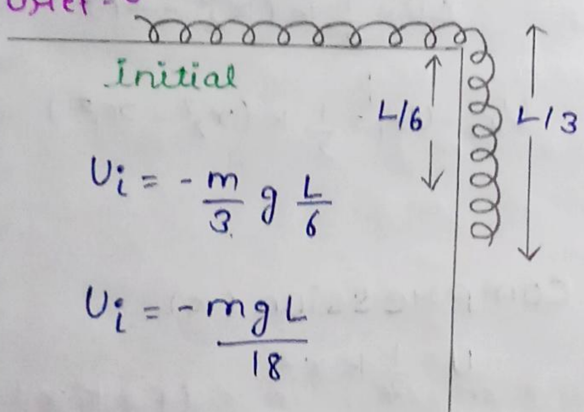


$U = U_1 + U_2 = 0 - \frac{m}{2} \times \frac{L}{4} g$

$U = -mg\frac{L}{8}$

Que- One 3rd Part of chain is hanging then find work to pull back the hanging part (slowly)

$U_{ref} = 0$



$U_i = -\frac{m}{3} g \frac{L}{6}$

$U_i = -\frac{mgL}{18}$

$W_{NCF} = \Delta V$

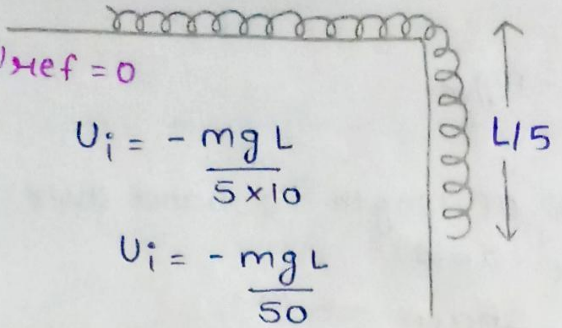
$= V_f - V_i$

$= 0 - \left[-\frac{mgL}{18}\right]$

$= \frac{mgL}{18}$

Que - A chain is on a frictionless table with one fifth of its length hanging over the edge. If the chain has length L and mass M, the work required to be done to pull the hanging part back onto the table is (very slowly)

Chain (M, L)



$U_{ref} = 0$

$$U_i = -\frac{mgL}{5 \times 10}$$

$$U_i = -\frac{mgL}{50}$$

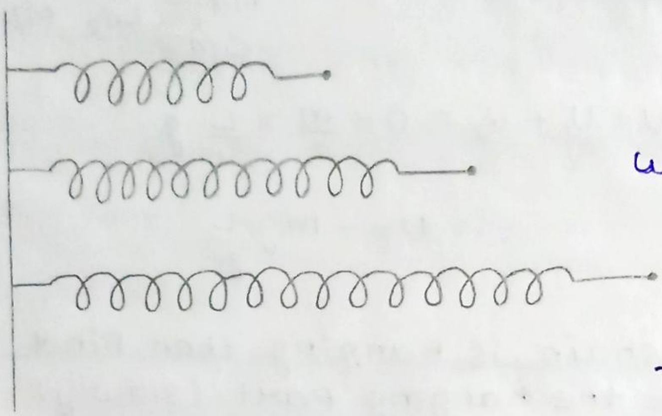
$$W_{N.C.F} = \Delta U$$

$$W_{N.C.F} = U_f - U_i$$

$$= 0 - \left(-\frac{mgL}{50}\right)$$

$$= +\frac{mgL}{50}$$

Work Done By Spring Force and Potential Energy



Work done by S.f in elongation from x_1 to x_2

$$W_{Sf} = -\frac{1}{2}K(x_2^2 - x_1^2) \quad \text{— (i)}$$

$$W_{C.F} = -\Delta U \quad \text{— (ii)}$$

$$-\Delta U = -\frac{1}{2}K(x_2^2 - x_1^2)$$

$$\Delta U = \frac{1}{2}K(x_2^2 - x_1^2)$$

$$U_{x_2} - U_{x_1} = \frac{1}{2}K(x_2^2 - x_1^2)$$

If $x_1 = 0$ (mean)

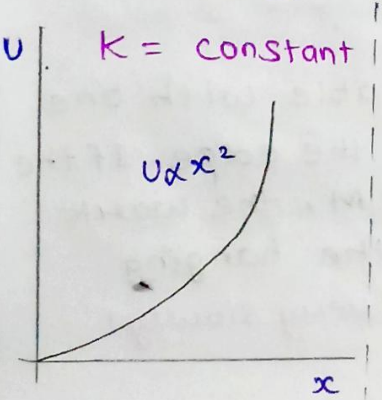
at mean ($U_{ref} = 0$) Let

$$U_x = \frac{1}{2}Kx^2$$

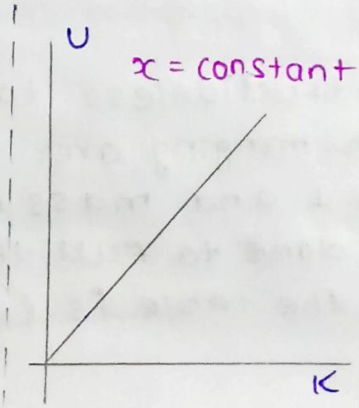
Stored P.E. at elongation / compression (x)

$$U = \frac{1}{2}Kx^2$$

K = constant



x = constant



$$U = \frac{1}{2}Kx^2$$

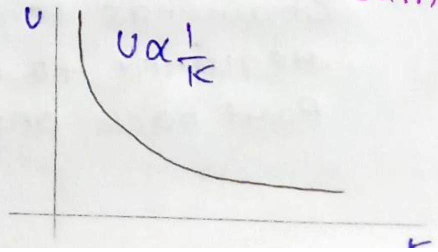
$$U = \frac{1}{2}K\left(\frac{F^2}{K^2}\right)$$

$(F = Kx)$

$x = \frac{F}{K}$

$$U = \frac{1}{2}\frac{F^2}{K}$$

F = same



Que- Force constants K_1 and K_2 of two springs are in the ratio 5:4. They are stretched by same length. If potential energy stored in one spring is 25 J then potential energy stored in second spring is

$$\frac{K_1}{K_2} = \frac{5}{4} \qquad \frac{U_1}{U_2} = \frac{\cancel{\frac{1}{2}} K_1 \cancel{x^2}}{\cancel{\frac{1}{2}} K_2 \cancel{x^2}} = \frac{5}{4}$$

$$\frac{25}{U_2} = \frac{5}{4} \rightarrow U_2 = 20\ J$$

Que- Two spring have their force constants in the ratio of 3:4. Both the springs are stretched by applying force F. If elongation in first spring is x then elongation in second spring is

$$\frac{K_1}{K_2} = \frac{3}{4}$$

$$F_1 = F_2$$

$$K_1 x_1 = K_2 x_2$$

$$\left(\frac{K_1}{K_2}\right) x_1 = x_2$$

$$\frac{3}{4} x = x_2$$

$$F = \text{constant}$$

$$K_1 x_1 = K_2 x_2$$

$$\frac{K_1}{K_2} x_1 = x_2$$

$$\frac{3}{4} x = x_2$$

Que - A spring with spring constant K_0 when compressed by 1cm, the potential energy stored is U. If it is further compressed by 3 cm, then change in its potential Energy

$$U = \frac{1}{2} K (1)^2 \longrightarrow U = \frac{K}{2}$$

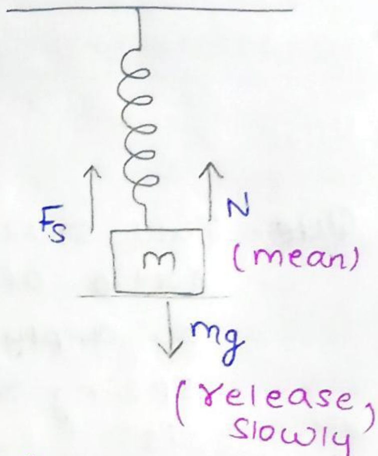
$x_i = 1\,cm \qquad x_f = 4\,cm$

$$\Delta U = \frac{1}{2}k(x_f^2 - x_i^2)$$

$$= \frac{1}{2}k[4^2 - 1^2]$$

$$\Delta U = U[16 - 1]$$

$$\Delta U = 15U$$



C.O.M.E. kaise apply karenge?

C.O.M.E is not applicable

जहा C.O.M.E. वहा हम नही
जहा हम वहा C.O.M.E. नही

$$\uparrow F_S + N \downarrow = mg \text{ (constant)}$$

$$N = 0$$

$$F_{S\,max} = mg$$

$$kx = mg$$

$$x = \frac{mg}{k}$$

Que- A particle of mass 'm' is moving in horizontal circle of radius r under centripetal force equal to $-k/r^2$; where k is constant then find total mechanical energy

$$F_c = \frac{mv^2}{r} = \frac{k}{r^2}$$

towards

$$F = +\frac{k}{r^2} = +\frac{dU}{dr}$$

$$\int \frac{k\,dr}{r^2} = \int dU$$

$\frac{1}{2}mv^2 = \frac{1}{2}\frac{K}{r}$

$K.E = \frac{K}{2r}$

$K\left[-\frac{1}{r}\right] = U$

$U = -\frac{K}{r}$

$T.E = K.E + U$

$= \frac{K}{2r} - \frac{K}{r} = \frac{K}{2r}$

Que- If M.E. = 4J , Find maximum speed (Kmax)

$U(x) = \frac{x^3}{3} - \frac{x^2}{2}$

U will be maxm/minm at

$\frac{dU}{dx} = 0$

$\frac{dU}{dx} = \frac{3x^2}{3} - \frac{2x}{2} = 0$

$x^2 - x = 0$

$x(x-1) = 0$

$\left[\begin{matrix} x=0 \\ x=1 \end{matrix}\right]$

$U_{(x=0)} = 0$

$U_{(x=1)min} = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$

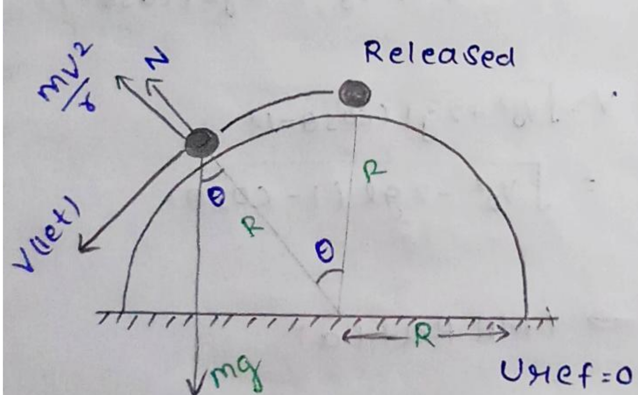
$K.E_{max} + U = 4J$

$KE_{max} + U_{min} = 4J$

$\frac{1}{2}mv^2_{max} + U_{min} = 4J$

$\frac{1}{2}mv^2_{max} - \frac{1}{6} = 4$

Que- A ball of mass m released then Find angle from vertical where, it will loose contact, hemispherical surface is smooth



Ramlal is sitting on object

$\frac{mv^2}{R} + \cancel{N}^{0} = mg\cos\theta$

To loose contact

$v^2 = Rg\cos\theta \quad \text{—(i)}$

(COME) b/w initial and final

$(KE+U)_i = (KE+U)_f$

$0 + mgR = \frac{1}{2}mv^2 + mgR\cos\theta$

$2(gR - gR\cos\theta) = v^2$ — (ii)

$Rg\cos\theta = 2Rg(1-\cos\theta)$

$\cos\theta = 2 - 2\cos\theta$

$3\cos\theta = 2 \rightarrow \cos\theta = \frac{2}{3}$

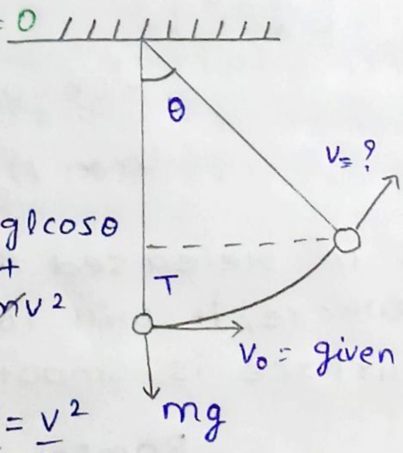
$\theta = \cos^{-1}\left(\frac{2}{3}\right)$

Verticle Circular Motion

Que- Find Velocity of ball when string will make an angle θ from vertical (m)

$W_{Tension} = 0$

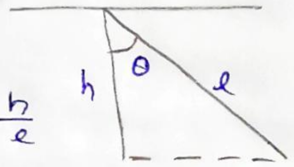
(COME) A & P



$\frac{1}{2}\cancel{m}v_0^2 - \cancel{m}gl = -\cancel{m}gl\cos\theta + \frac{1}{2}\cancel{m}v^2$

$\frac{v_0^2}{2} + gl\cos\theta - gl = \frac{v^2}{2}$

$\frac{v_0^2}{\cancel{2}} + 2gl(\cos\theta - 1) = \frac{v^2}{\cancel{2}}$



$\cos\theta = \frac{h}{l}$

$h = l\cos\theta$

MR*

If $\theta = 0°$

$V = V_0$

$V^2 = V_0^2 + 2gl(\cos\theta - 1)$

$V = \sqrt{V_0^2 + 2gl(\cos\theta - 1)}$

$= \sqrt{V_0^2 - 2gl(1-\cos\theta)}$

$V = \sqrt{V_0^2 + 2gl(\cos\theta - 1}$

↳ θ ↑ = speed ↓

$V = \sqrt{V_0^2 - 2gl(1-\cos\theta)}$

Que- Find relation b/w H & R so that Object will complete vertical circular motion

$H \geq R$ ×

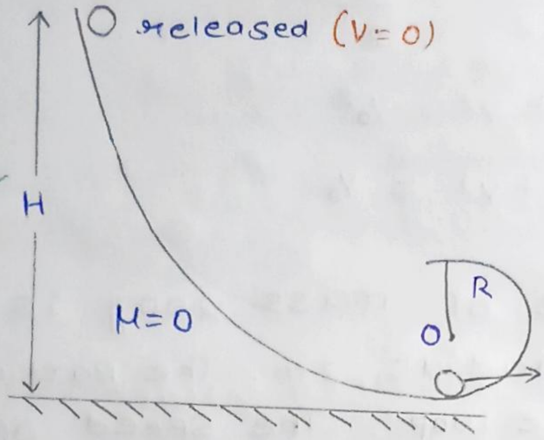
मिला < चाहिए ×

मिला ≥ चाहिए ✓

$\sqrt{2gH} \geq \sqrt{5gR}$

$2H \geq 5R$

$H \geq \frac{5R}{2}$



$U_{gain} = \sqrt{2gH}$

$U_{required} = \sqrt{5gR}$ ✓ (चाहिए)

Que- A stone of mass 1 kg is tied with a string and it is whirled in a vertical circle of radius 1m. If tension at the highest point is 14 N, then velocity at lowest point will be

$$T_\theta = \frac{mV_0^2}{l} - 2mg + 3mg\cos\theta$$

$$14\,N = \frac{V_0^2 \times 1}{1} - 2mg + 3mg(\cos 180^\circ)$$

$$14\,N = V_0^2 - 5 \times mg$$

$$50 + 14 = V_0^2$$

$$V_0 = \sqrt{64} = 8\,m/s$$

Que- A stone is tied to one end of a light inextensible string of length l and made to rotate on a vertical circle keeping other end of the string at the centre. If speed of the highest point is v ($v > \sqrt{gl}$) then its speed at the lowest point is

COME

$$mgl + \frac{1}{2} mv^2 = \frac{1}{2} mV_0^2 - mgl$$

$$2 \times 2gl + \frac{v^2}{2} = \frac{V_0^2}{2}$$

$$4gl + v^2 = V_0^2$$

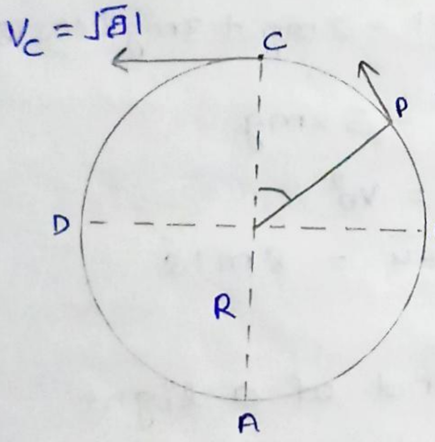
$$\sqrt{4gl + v^2} = V_0$$

Que- A particle of mass 200g is moving in a circle of radius 2m. The particle is just "looping the loop". The speed of the particle and the tension in the string at highest point of circular path are ($g = 10 m/s^2$)

$$V = \sqrt{gl} = \sqrt{10 \times 2} = \sqrt{20} = 4.47 N$$

Que- A particle is moving along a vertical circle of radius R. The velocity of particle at P will be : (assume critical condition at c)

(COME)$_{P\&C}$



$$\frac{1}{2} mV^2 + mgR\cos\theta = \frac{1}{2} m(\sqrt{gR})^2 + mgR$$

$$\frac{V^2}{2} = -gR\cos\theta + \frac{gR}{2} + gR$$

$$\frac{V^2}{2} = -gR\frac{1}{2} + \frac{gR}{2} + gR$$

$$V = \sqrt{2gR}$$

Given velocity	Compt. circular Motion	Tension at top	V at top
$V > \sqrt{5gl}$	yes	$T > 0$	$V > \sqrt{gl}$
$V = \sqrt{5gl}$	Just comp. loop yes	$T = 0$	$V = \sqrt{gl}$
$\sqrt{2gl} < V < \sqrt{5gl}$	NO	will leave circular Path when ($\theta > 90°$) where T will be zero.	After leaving circular path then it will move on Parabolic
$V < \sqrt{2gl}$	NO	will leave circular Path when $\theta < 90°$, where velocity become zero	Oscillation
$V <<< \sqrt{2gl}$	NO	"	simple Harmonic Motion

Que- If velocity $V = \sqrt{3gR}$ is given at the mean Position then Find angle where bob will leave circular Path

$\theta > 90°$ will leave circular path where $T = 0$

$\theta = \cos^{-1}\left(-\frac{1}{3}\right)$

$$T = \frac{mV_0^2}{l} - 2mg + 3mg\cos\theta$$

$$0 = \frac{m\,3gR}{l} - 2mg + 3mg\cos\theta$$

$$mg + 3mg\cos\theta = 0$$

$$\cos\theta = -\frac{1}{3}$$

Que- If given velocity at mean is $V = \sqrt{gR}$ then angle where it will leave circular Path

Circular Path will leave where velocity became zero

$$V^2 = V_0^2 + 2gl(\cos\theta - 1)$$

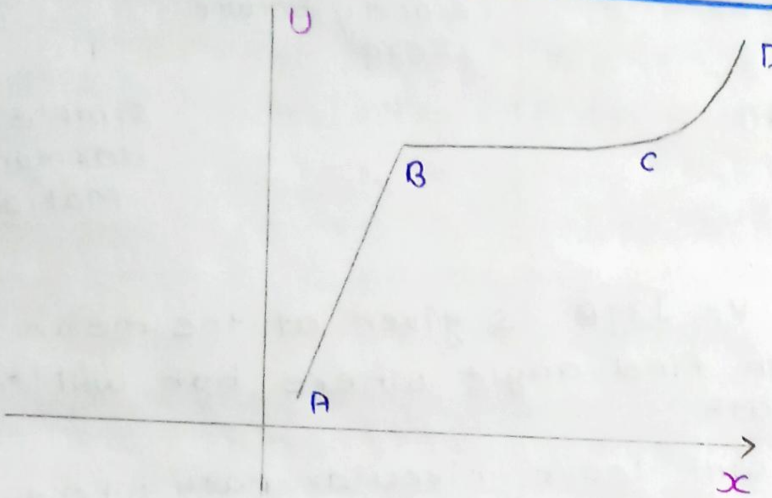
$$gR + 2gR(\cos\theta - 1) = 0$$

$$gR + 2gR\cos\theta - 2gR = 0$$

$$2gR\cos\theta = gR$$

$$\cos\theta = \frac{1}{2} \qquad \theta = 60^\circ$$

Potential Energy Distance Graph

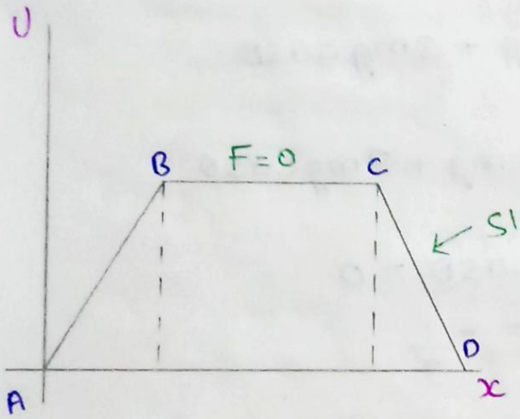


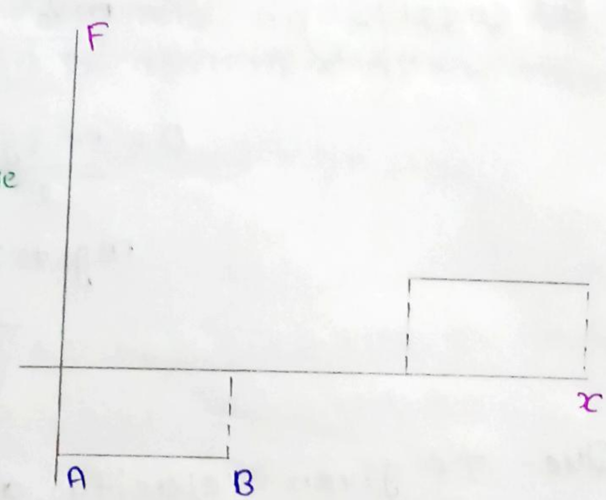
$\vec{F}_{C.F} = -\underbrace{\frac{dU}{dx}}_{\text{slope}}$

$\vec{F}_{AB} = -\text{Ve const.}$

$\vec{F}_{BC} = \text{zero}$

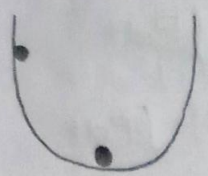
$F_{CD} = (+\text{ve})\uparrow = -\vec{F}\uparrow$ (inc.)



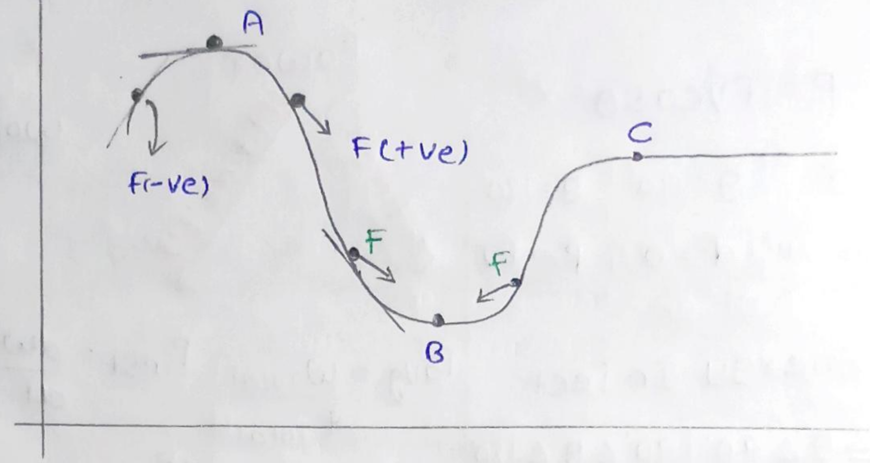


Force = - (slope (u/x))

U/x Graph



min → stable



At 'A'

$$F = -\frac{dU}{dx} = 0$$

$$\frac{d^2U}{dx^2} = -ve$$

Unstable eqm

At 'B'

$$\vec{F} = -\frac{dU}{dx} = 0$$

equilibrium

- Stable equilibrium

$$\frac{d^2U}{dx^2} = +ve$$

minima

At 'C'

$$F = 0$$

$$\frac{d^2U}{dx^2} = 0$$

Neutral eqm

(Area of (ω-t) graph)

$\Delta K.E = W = \int P dt$ = Area under Power-time graph

$\int W dt$

नाली ← (W) ← P (Power) → नाली

diff $\left(\frac{dW}{dt}\right)$

diff.

Slope (W/t) graph

Power

$P = FV\cos\theta$

$\theta = 0°$	$\theta = 90°$	$\theta = 180°$
$P_{max} = FV$	$P = 0$	$P = -FV$

Energy Inject

$0 \le \theta < 90°$	$90° < \theta \le 180°$
$P = +ve$	$P = -ve$

Power

(In time 't') (At time 't')

$\int dw = \int P\,dt$

$work = \Delta K.E = \int P\,dt$

$P_{avg} = \frac{W_{Total}}{t_{Total}}$

$P_{avg} = \frac{\Delta K.E.}{time}$

$P_{avg} = \frac{\int P\,dt}{\int dt}$

$P_{inst} = \frac{dw}{dt}$ = The Rate of work done w.r.t time

- P_{inst} = Slope of work time graph
- $P_{inst} = \frac{dw}{dt} = \frac{\vec{F}.dx}{dt} = \vec{F}.\vec{u}$ $\boxed{dw = f.dx}$
- $P_{inst} = \vec{F}.\vec{V} = FV\cos\theta$

Scalar = J/s = watt

θ → Angle b/w Force & Velocity

Que- One coolie takes 1 minute to raise a suitcase through a height of 2m but the second coolie takes 30 s to raise the same suitcase to the same height. The powers of two coolies are in the ratio

$P_{avg_1} = \frac{mgs}{t} = \frac{mgs}{60\ sec}$

$P_2 = \frac{mgs}{t} = \frac{mgs}{30\ sec}$

$\frac{P_1}{P_2} = \frac{30}{60} = \frac{1}{2}$

Que- If force of 9N is acting on a body then find instantaneous power supplied to the body

When its velocity is 5 m/s in the direction of Force

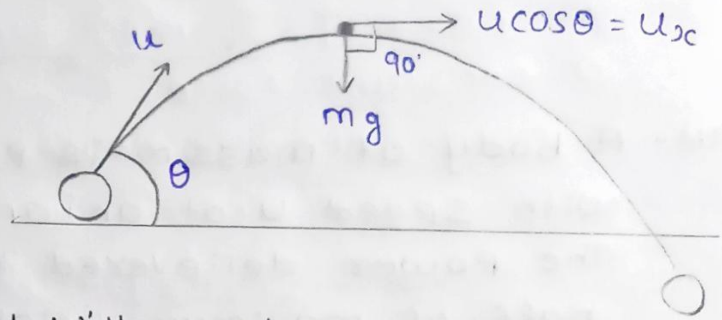
$$P = \frac{dw}{dt} = \vec{F}\cdot\vec{u}$$

$$= Fv\cos\theta$$

$$P = 9 \times 5 \cos 0^\circ = 45\, W$$

Que - Ball is Projected u at angle θ then find Power delievered by gravitational force at maxm height

$F_{inst} = mg\, u_x \cos 90^\circ = 0$

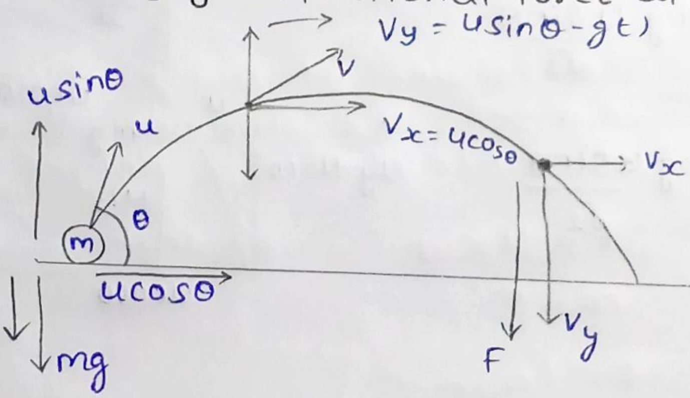


Que - Ball is Projected with u at angle θ then find Power delievered by gravitational Force at time 't'

$P = -mg\,(u\sin\theta - gt)$

t = 0

at a time of Projection $P = -mg\,u\sin\theta$



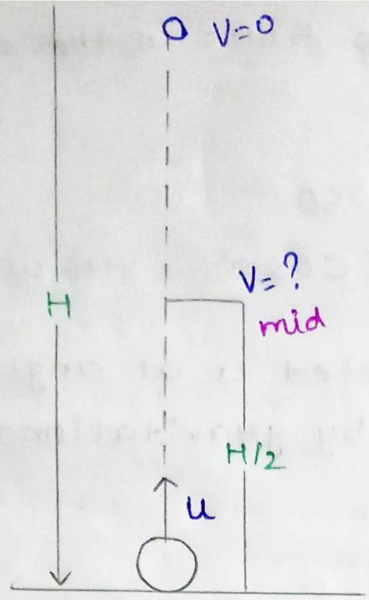
MR *

$t = \frac{u\sin\theta}{g} \rightarrow P_{at\ max\ height} = 0$

$a = \text{constant}$

$$u_{mid} = \sqrt{\frac{u^2 + v^2}{2}}$$

Valid for 1-D Motion



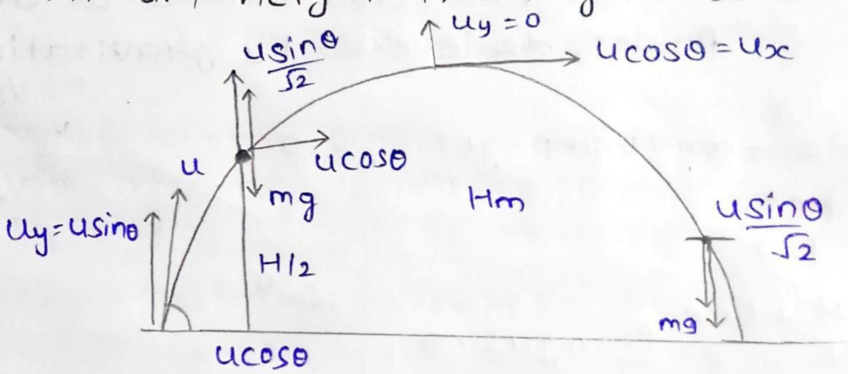
$$V_{mid} = \sqrt{\frac{u^2 + 0}{2}}$$

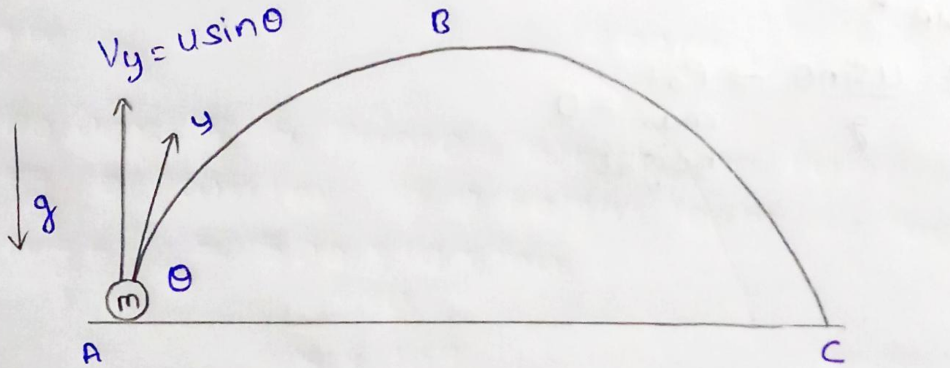
$$u_{mid} = \frac{u}{\sqrt{2}}$$

Que- A body of mass m is projected from ground with speed u at an angle θ with horizontal. The power delievered by gravity to it at half of maximum height from ground is

$$P = -mg\frac{u\sin\theta}{\sqrt{2}}$$

$$P = mg\frac{u\sin\theta}{\sqrt{2}}$$



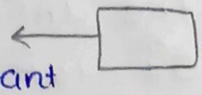


Power delievered by gravitational force from A to B is -ve (True)

Power delievered by gravitational force from B to C is +ve (True)

Magnitude of power delievered is decreaseas object move from A to B (True)

Que - A body is being moved from rest along a straight line by a machine delivering constant power. The distance covered by body in time t is proportional to



P = constant

$P = FV$

$P = \underbrace{ma}_{\text{constant}}V$

$aV = \text{constant}$

$\frac{dV}{dt}V = \text{constant}$

$\int V dV = K \int dt$

$\frac{V^2}{2} = Kt$

$V \propto \sqrt{t}$

$\frac{dx}{dt} = \sqrt{t}$

$P_{inst} = P_{Avg}$

P = constant

$\int dx = \int \sqrt{t}\, dt$

$x = \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} \propto t^{3/2}$

Que - If position of object $x \propto t$ then find power delievered as a function of time

$x \propto t$

↳ diff. w.r.t 't'

$V = \frac{dx}{dt} \propto 1$ (constant)

$\frac{dV}{dt} = a = 0$

$P_{Inst} = FV$

$= maV$

$P_{Inst} = 0$

Que - The position (x) of a body moving along x-axis at time (t) is given by $x = 3t^2$ where x is in metre and t is in second. If mass of body is 2 kg, then find the instantaneous power delievered to body by force acting on it at t = 4 s

$x = 3t^2$ $\longrightarrow$ $\frac{dx}{dt} = 3(2t) = 6t$ — (i)

$m = 2kg$

$V = 6t$

$\hookrightarrow \frac{dV}{dt} = a = 6$

$P = \frac{dw}{dt} = FV = maV$

$= 2 \times 6 \times 6t$

$(P) = 72t \rightarrow P = (72 \times 4)\,w = 288$

Que- A particle moves with the velocity $\vec{V} = (5\hat{i} + 2\hat{j} - \hat{k})\ ms^{-1}$ under the influence of a constant force, $\vec{F} = (2\hat{i} + 5\hat{j} - 10\hat{k})\,N$. The instantaneous power applied

$\vec{u} = 5\hat{i} + 2\hat{j} - \hat{k}$

$\vec{F} = 2\hat{i} + 5\hat{j} - 10\hat{k}$

$\vec{P} = \vec{F}.\vec{u} = 10 + 10 + 10 = 30w$

Que- The power of water pump is 4kw. If $g = 10\ ms^{-2}$, the amount of water it can raise in 1 minute to a height of 20 m is

$P = 4kw$

$m = ?$

$t = 1\ min$

$h = 20m$

$P = \frac{mgh}{t}$

$4 \times 10^3 = \frac{m \times 10 \times 20}{60}$

$12 \times 10^3 = 10m$

$1200m$

Que- The power of a pump, which can pump 500 kg of water to a height 100 m in 10s is

$P = \frac{mgh}{t} = \frac{500 \times 10 \times 100}{10}$

$= 5 \times 10^4$

$= 50 \times 10^3$

$= 50\ kw$

Que- An engine pumps 800 kg of water through height of 10 m in 80 s. Find the power of the engine if its efficiency is 75%.

$m = 800\text{ kg}$

$h = 10\text{ m}$

$t = 80\text{ s}$

$P = ?$

$\eta = 75\%$

MR*

$$P = \frac{mgh}{t}$$

$$P_{pump} \times 75\% = \frac{mgh}{t}$$

$$P \times \frac{75}{100} = \frac{800 \times 10 \times 10}{80}$$

$$P \times \frac{75}{100} = 10^3$$

$$P = \frac{4}{3} \times 1000$$

$$= \frac{4}{3}\text{ kW}$$

Que- A car of mass m starts from rest and accelerates so that the instantaneous power delievered to the car has a constant magnitude P_0. The instantaneous velocity of this car is proportional to

CAR

$u = 0$

$P_0 = \text{constant}$

(constant) $P_0 = mav$

$$P_0 = m\frac{dv}{dt}v$$

$$\int P_0\,dt = \int mv\,dv$$

$$P_0 t = \frac{mv^2}{2}$$

$$v^2 \propto t$$

$$v \propto \sqrt{t}$$